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COMMENT

On the non-existence of linear non-reciprocal bi-isotropic (NRBI) media

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Abstract. We show that Raab and Sihvola's theses on the recognizable existence of linear nonreciprocal bi-isotropic media are either incorrect or inconsequential to modern electromagnetic theory, which has purely microscopic foundations.

1. Introduction

This communication is in response to the extensive comments made by Raab and Sihvola [1] (hereafter denoted as RS). These comments relate specifically to a letter [2] by the first author of this response and, more generally, to our work on a constraint on the constitutive relations of linear electromagnetic material media. Perhaps the most important by-product of this constraint is the negation of the recognizable existence of the so-called non-reciprocal bi-isotropic (NRBI) media.

As the specific constraint was originally given by Post [3], though we later contextualized it with respect to the microscopic basis of electromagnetism [4–6], we call it the Post constraint (PC) here. We must also mention that our emphasis has been all along on the *recognizable existence* of NRBI materials, not simply on the *existence* thereof: an issue such as the *non-recognizable existence* of certain entities lies beyond our scope.

RS advances two major theses:

(1) The PC does not emerge from the structure of modern electromagnetics, but is a symmetry constraint instead.

(2) Materials responding to time-independent fields can be bi-isotropic.

In this response, we show that the first thesis is incorrect, the second inconsequential.

2. Covariance and uniqueness versus symmetry

In order to keep to the same level of complexity as RS, let us consider a linear, homogeneous, bi-anisotropic medium with time-invariant (not time-independent) response properties. The time-domain constitutive relations of this medium are compactly stated as the convolution integral

$$G_{kl}(t, \boldsymbol{x}) = \frac{1}{2}\int_{-\infty}^{\infty} \chi_{klnm}(\tau) F_{mn}(t-\tau, \boldsymbol{x}) \tag{1}$$

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where all quantities are purely real-valued; $G_{kl}(t, \boldsymbol{x})$ are the components of a skew-symmetric tensor of rank 2 comprising the induction fields $\boldsymbol{D}(t, \boldsymbol{x})$ and $\boldsymbol{H}(t, \boldsymbol{x})$; $F_{mn}(t, \boldsymbol{x})$ are the components of a skew-symmetric tensor of rank 2 comprising the primitive fields $\boldsymbol{E}(t, \boldsymbol{x})$ and $\boldsymbol{B}(t, \boldsymbol{x})$; and $\chi_{klnm}(t)$ are the components of the constitutive property tensor of rank 4, with $\chi_{klnm}(t < 0) \equiv 0$. In addition, $\chi_{klnm}(0) = \chi_{klnm}^{\text{vacuum}}(0)$ due to the microscopic nature of modern electromagnetics [7, 8]: thus, all *materials* whose instantaneous response properties are different from that of vacuum (i.e. free space) must be excluded from (1) because microscopic material particles cannot respond instantaneously. The non-quantum (i.e. classical) field description has been adopted here, in keeping with RS and the papers cited earlier [2–7].

Now, Post proved in section VI.2 of his book that the 'alternating components in the constitutive [property] tensor ... lead to an identically vanishing contribution in the Euler–Lagrangian derivative', thereby giving rise to the PC:

$$\chi_{[klmn]}(t) = 0 \tag{2}$$

see [9] for a definition of the *alternation* of a tensor. Thus, the PC emerged as a *covariance* constraint! More than 30 years later, we [10] showed that the identical relation

$$\epsilon_{klmn}\chi_{klmn}(t) = 0 \tag{3}$$

emerges also as a *uniqueness* constraint mandated by the principle of parsimony, ϵ_{klmn} being the components of the fourth-rank permutation tensor.

Clearly, the PC cannot be a symmetry constraint for bi-anisotropic media because neither (2) nor (3) addresses the consequences of interchanging certain indices of $\chi_{klmn}(t)$. All that the PC states is that the weighted sum of certain components of the constitutive property tensor equals zero. Therefore, we must conclude that the deduction in RS that *the PC is a symmetry constraint* is incorrect.

Parenthetically, that the PC appears to be a symmetry constraint for bi-isotropic media is not more than pure coincidence. Indeed, it also turns out that for specializations such as bi-isotropic and certain types of uniaxial bi-anisotropic media [11]—but *not* for general bi-anisotropic media—the PC becomes a reciprocity constraint.

3. Time-independent fields

Let us remark that RS does not contain any suggestion that the proofs of the PC are erroneous. Instead, RS advance 'a different approach', discussing in detail the electromagnetic response properties of the so-called magnetic crystals. Indeed, RS claims that 'magnetic cubics (*sic*) with [certain] point group symmetries ... respond bi-isotropically to uniform electric and magnetic fields', and gives evidence for that statement from various publications†.

First, let us emphatically state that time-independent (i.e. uniform) electric and magnetic fields do not exist. The oft-claimed example of the electrostatic field between the plates of a parallel plate capacitor is misleading because a capacitor can never be charged fully, so that the electric field between the plates has to be time varying. Every time-varying electric field is accompanied by a magnetic field howsoever small, but not zero, in magnitude.

† Parenthetically, the mathematical formalism of RS uses a mixture of time dependence and frequency dependence: see equation (5) of RS, for example, where the constitutive relations are analysed in terms of *phasors* characterized by an angular frequency ω, yet, at the same time, formal time derivatives of the *fields* are also retained. This mixture obscures certain significant and relevant issues.

Second, all publications—theoretical as well as experimental—cited in RS in the particular context of time-independent fields are based on a premise we consider *microscopically unphysical*: that certain materials can respond instantaneously to applied electromagnetic fields, with their instantaneous response properties different from vacuum's. Whereas such materials are admissible in pre-electron *continuum electromagnetics*, the microscopic basis of modern electromagnetics does not allow such material media. This is because of the post-electron paradigm that matter is really a suspension of elementary discrete entities, loosely called particles, in vacuum. O'Dell [12] ignored this fact completely when he set $\chi_{klmn}(t > 0) = 0$ and entertained the microscopically unphysical possibility $\chi_{klmn}^{\text{material}}(0) \neq \chi_{klmn}^{\text{vacuum}}(0)$. His error was carefully dissected by one of us [13] some two years ago.

So, we have no objection to Raab and Sihvola's claim that there could exist 'magnetic (or nonreciprocal) [material] media whose constitutive relations for uniform electric and magnetic fields are bi-isotropic'. We grant this concession readily because their claim is *inconsequential* as a physical sample of that material cannot be produced; $\chi_{klmn}^{\text{material}}(0) \neq \chi_{klmn}^{\text{vacuum}}(0)$ is inadmissible in modern electromagnetics, which has a microscopic viewpoint [7, 8]. What can be produced are samples of materials—the magnetic cubics discussed in RS—that were incorrectly characterized previously on the basis of an unphysical theory that does not properly consider the microscopic nature of matter.

4. Conclusion

Having thus dispelled the chief arguments advanced in RS, we turn our attention briefly to the so-called Tellegen materials. These materials were conceptualized by Tellegen [14] nearly half a century ago on a purely speculative basis without the backup of any thorough analysis. We have often suggested in the past to the proponents of Tellegen materials that the simplest way to prove their existence is to follow Tellegen's recipe and attempt to glue ferromagnetic particles to electret microcrystals. The realization of physical samples of Tellegen materials in that way, followed by experimental characterization of their (causal) response characteristics, would make it unnecessary for their proponents to invoke unphysical entities such as time-independent fields and instantaneously responding materials. Alternatively, if possible, it must be shown correctly† that (2) and (3) were incorrectly obtained. At the present time, though, the objections to the PC remain unconvincing—on physical as well as mathematical grounds.

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† Incidentally, while local application of generalized duality in electromagnetics [15] is improper unless immediately reversed, global application of this principle is perfectly appropriate and does not negate the PC [16].

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